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The Study of Atomic Nuclei with Low and Medium Energy Neutrons

By S. E. Darden*

I. *Introduction.* In the last two decades the study of the interaction of neutrons with nuclei has played an important role in the investigation of nuclear structure. After the neutron was discovered in 1932 and it was realized that neutrons could be produced artificially in nuclear reactions, it became evident that one way of studying the nucleus would be to observe what happens when nuclei were bombarded by neutrons. Since the neutron possesses no electric charge, it can penetrate a nucleus much more easily than a charged particle, such as a proton or an alpha particle, which must overcome an appreciable electrostatic repulsion. A nuclear reaction in which neutrons are produced can usually be represented symbolically by the expression

$$a + A \rightarrow B + n + Q. \qquad (1)$$

In words this means that when the nucleus A is bombarded with the particle a, then a neutron n may be produced, leaving the residual nucleus B. The term Q on the right hand side refers to the kinetic energy released or absorbed in the reaction. If Q is positive, then the reaction is said to be exoergic; if Q is negative, the reaction is endoergic and cannot take place unless the particle a strikes the nucleus A with a kinetic energy equal to or greater than Q. The particle a can be a nuclear particle such as a proton, a light nucleus such as a deuteron or an alpha particle, or it may be a gamma ray. Typical neutron producing reactions which have turned out to be useful in the studies of nuclei are the $Li^7(p, n)Be^7$ reaction and the $D^2(d, n)He^3$ reaction. In the former, a target of lithium is bombarded with protons having an energy of 1,88 million electron-volts (MeV) or more, yielding neutrons and the residual nucleus Be^7. The $D^2(d, n)He^3$ reaction on the other hand is exoergic, with a Q of almost 4 MeV. As a rule, the charged particles initiating the reaction come from a particle accelerator such as an electrostatic generator, a Cockroft-Walton machine or a cyclotron. The neutron energy can be varied by changing the energy of the bombarding charged particle or the angle at which the neutrons are observed, measured with respect to the direction of the charged particle beam.

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In order that use may be made of neutron beams in the study of nuclei, some means for detecting the neutrons must be employed. Since the neutron possesses no electric charge, it produces by itself no ionization in passing through matter and is therefore somewhat less easy to detect than a charged particle. In order to be susceptible to detection, the neutron must interact with other nuclei. This interaction may either be a nuclear reaction yielding one or more charged particles or it may be an elastic scattering event, in which the recoiling charged particle receives an appreciable amount of the neutron's kinetic energy. In either case, the recoiling charged particle ionizes atoms of the material through which the neutrons are passing and the subsequent measurement of this ionization constitutes the detection of the neutron. Since the amount of ionization produced is proportional to the energy of the recoiling charged particle, it is desirable that the latter be as energetic as possible. In the case of an elastic collision, this requires that the struck particle be as light as possible, since the lighter the particle, the more recoil energy it takes up in the collision. For this reason hydrogen is the most commonly used substance in recoil detectors.

Two frequently employed detectors of this sort are the hydrogen recoil counter and the organic scintillator detector. The former consists of a chamber filled with hydrogen or some hydrogen-containing gas. An electrode is placed in the chamber and held at some potential with respect to the counter walls. When a recoil proton ionizes the gas, the electric field causes the ions and electrons to move in opposite directions, and this movement induces a voltage pulse on the electrode. The pulses are then amplified and counted.

The organic scintillator detector makes use of a solid or liquid organic material into which is mixed a scintillating substance. Ionization of the scintillator molecules by the recoiling protons gives rise to light flashes. These light flashes are then detected by a photomultiplier tube, a device similar to a photoelectric cell, but having a large amplification brought about by secondary electrons. Both the recoil counter and the scintillation detector have found extensive application in neutron experiments. Their use is of course limited to the detection of neutrons possessing sufficient energy to enable the ionization produced to be detectable above the electronic noise of the amplifier.

The interaction of a beam of neutrons with a given nucleus is most conveniently described in terms of cross sections. For a given process, the cross section can be defined as the number of times the process occurs per unit time per target nucleus per unit neutron flux incident on the target. The neutron flux is defined as the number of neutrons crossing unit area perpendicular to the direction of the beam per unit time. From this definition it is clear that the units of cross section are the units of area, and the unit generally used is the barn, which is defined as 10^{-24} cm^2. The cross section for a given process can be considered to be the area which the nucleus presents to a neutron as far as the process in question is concerned. If one adds together the cross sections for all the processes which may occur when a given nucleus is bombarded with neutrons, one obtains the total cross section, usually designated σ_t. One might then expect total cross sections to be of the order of magnitude of the square of nuclear dimensions, or about $(10^{-12}\,\text{cm})^2 = 10^{-24}\,\text{cm}^2 = 1$ barn, and indeed the measured total cross sections of nuclei are, generally speaking, of the order of a few barns.

It turns out to be convenient to break the total cross section down into cross sections for various processes which can occur. One such partial cross section is the differential elastic scattering cross section $\sigma(\vartheta)$. This represents the probability that a neutron will be scattered without loss of energy, i.e. elastically, through an angle ϑ, where ϑ is measured with respect to the direction of the incident neutron beam. Upon summation of $\sigma(\vartheta)$ over the entire solid angle, one obtains the total elastic scattering cross section, σ_{el}. This can be written

$$\sigma_{el} = \int \sigma(\vartheta)\; d\Omega, \qquad (2)$$

where $d\Omega$ is the differential unit of solid angle and the integration is extended over the entire solid angle. The difference between the total cross section and the total elastic scattering cross section is referred to as the non-elastic or inelastic collision cross section, σ_{in}, and is simply the cross section for all processes other than elastic scattering. Although a separation of σ_{in} into cross sections for various non-elastic processes is useful when one wishes to study some particular process, such as a nuclear reaction (for example, the reverse of one of the neutron-producing reactions mentioned above), this paper will be primarily concerned with the total, differential and non-elastic cross sections.

Another characteristic of the neutron-nucleus scattering interaction useful in studying the nucleus is the polarization produced in a beam of neutrons undergoing a nuclear scattering. As a consequence of the fact that the neutron possesses an intrinsic angular momentum, called the spin, it is possible for a beam of neutrons to exhibit a polarization in that the spins, instead of being randomly oriented, point, on the average in some particular direction. This definition is somewhat loose and qualitative, as the spin is a purely quantum mechanical property of particles. In view of the wave-like nature of atomic and subatomic particles, a neutron beam can be represented mathematically as a plane wave propagating in the direction the neutrons are moving. This plane wave can then be decomposed into a sum of waves, each of which corresponds to a definite orbital angular momentum of the neutron with respect to the scattering nucleus. If the scattering interaction is dependent upon the relative orientation of the neutron spin and the orbital angular momentum of the neutron relative to the nucleus, then it is possible for the neutron spin to be reoriented during the collision, and the interference between the reoriented part and the undisturbed part of the scattered neutron wave can lead to a polarization of the scattered beam. If the extent of this polarization can be measured, information concerning the scattering interaction may be obtained.

Two important advances in the theory of the interaction of neutrons with nuclei were made in the year 1936 by the Danish physicist Bohr[1] and the American physicists Breit and Wigner[2]. Bohr proposed that the interaction of neutrons with a nucleus involved the formation of the so-called compound nucleus. Such a situation would arise whenever the neutron strikes the nucleus and is absorbed by it, forming a compound system. One may picture this classically by imagining a nucleus to consist of a cluster of neutrons and protons held together by the nuclear forces. When the neutron strikes this assembly, it rapidly exchanges its energy with the constituent particles and is quickly amalgamated into the system as a whole. This compound system, which is unstable, can then decay by emission of nuclear particles, gamma rays or in some cases, as was discovered later, by nuclear fission. Since the compound nucleus is a quantum-mechanical system of interacting particles, it is expected to possess discrete energy levels, somewhat in the way an atom does. However since the system is unstable against the emission of a nuclear particle, the states are only quasi-stationary, again in analogy with the atom. It was in connection with these states that Breit and Wigner made their contribution. They developed formulas giving the cross sections for elastic scattering and absorption of neutrons having energies corresponding to one of these quasi-stationary states. For certain very simple cases the total cross section according to the Breit-Wigner formula may be written

$$\sigma_t = \sigma_0 + \pi \lambda\!\!\!{}^{-2}\, \frac{(2J+1)}{2\,(2I+1)} \cdot \frac{\Gamma n^2}{(E-E_r)^2 + \Gamma n^2/4} \cdot \qquad (3)$$

[1] N. Bohr, Nature *137*, 344 (1936).
[2] G. Breit and E. P. Wigner, Phys. Rev. *49*, 519, 642 (1936).

E is the kinetic energy of the incident neutrons, and $\lambda\!\!\!^-$ is the associated De Broglie wavelength. E_r is a constant energy characteristic of the quasi-stationary state, and is referred to as the resonance energy. I is the spin angular momentum of the target nucleus, while J is the total angular momentum of the compound state. The quantity Γn is a characteristic of the resonance and is called the neutron width. σ_0 represents the cross section away from the resonance, or potential scattering cross section as it is called. Equation (3) is plotted as a function of neutron energy in Figure 1. It can be seen that the presence of a resonance state greatly increases the probability of interaction for neutrons having energies near the resonance energy. The height of the peak in the cross section is related to the total angular momentum of the system in the compound state. In the case represented in Figure 1 the width of the resonance at half-maximum is just equal to the neutron width. Although this is not generally true, the width at half-maximum frequently gives an indication of Γn and therefore some qualitative information about the compound nuclear state[3]. It is shown in the theory of nuclear reactions[4] that Γn is proportional to the probability that the compound nucleus in the resonant state can decay into a neutron and a target nucleus in its ground state, i.e. just the reversal of the prooess involved in forming the compound state. Accordingly a very narrow resonance indicates a small probability for decay with the result that the state is relatively long-lived. Another way of looking at this is to consider the Heisenberg uncertainty relation $\Delta E\,\Delta t \sim h$, where h is PLANCK's constant. If the width of the resonance is associated with the energy uncertainty ΔE, then the life-time Δt of the state can be estimated from this relation. For example, a resonance having a width of 1000 eV will have a lifetime of the order of

$$h/\Delta E = \frac{6\times 10^{-27}\ \text{ergs}}{1000\ \text{eV}} = \frac{6\times 10^{-27}\ \text{erg s}}{1000\times 1{\cdot}6\times 10^{-12}\ \text{erg}} \simeq 10^{-18}\text{s}.$$

Such compound nuclear states are thus extremely short-lived, compared with the excited states of atoms for example, which usually have life times of the order of 10^{-8} s.

From these considerations it is evident that measurements of interaction cross sections using neutron beams of sufficiently well defined energy may yield curves of the sort shown in Figure 1. From a comparison of such data with the Breit-Wigner formulas, information about the resonant states of the compound nucleus may be obtained. The study of nuclear energy levels in this way forms part of the broad field of nuclear spectroscopy.

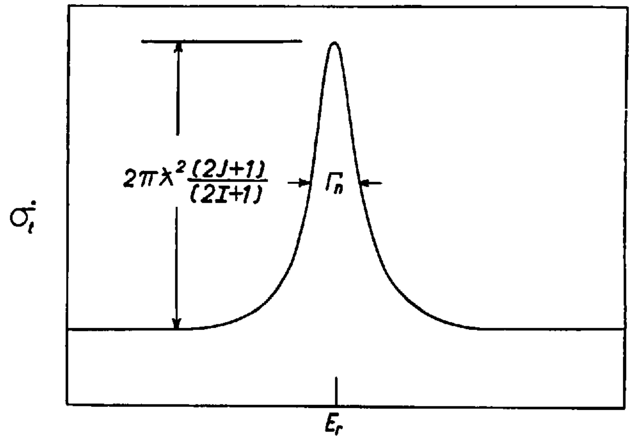


Fig. 1.—Total cross section in the neighborhood of a resonance according to equation (3). σ_t is plotted as a function of neutron bombarding energy.

In the next section of this paper the application of the methods outlined above to the study of energy levels in light nuclei is discussed.

II. *Light Nuclei.* By the term light nuclei is generally understood nuclei having atomic weights up to about 20. In such nuclei the number of constituent particles is relatively small, and it is to be expected that the level structure is accordingly rather more simple than in heavy nuclei. This supposition has been shown to be true in a number of ways, among them being measurements in good resolution of neutron total cross sections for light nuclei. The level structure of the compound nucleus revealed by such measurements is generally much less complicated than for heavy nuclei, where the resonances turn out in most cases to be so close together that they are difficult, when not impossible, to resolve[5].

The experimental study of light nuclei with neutrons has concentrated on the determination of the total cross sections[6] σ_t, and the differential elastic scattering cross sections $\sigma(\vartheta)$. The methods for measuring these cross sections are in principle very simple[7]. In order to determine the total cross section of an element, one needs only to measure the transmission of a beam of neutrons passing through a known thickness of a sample of the element. The transmission is then given in terms of the total cross section by the expression

$$T = e^{-nl\sigma_T}, \tag{4}$$

where n is the number of nuclei per cubic centimeter and l is the length of the sample in centimeters. In order to measure σ_t in good resolution it is necessary

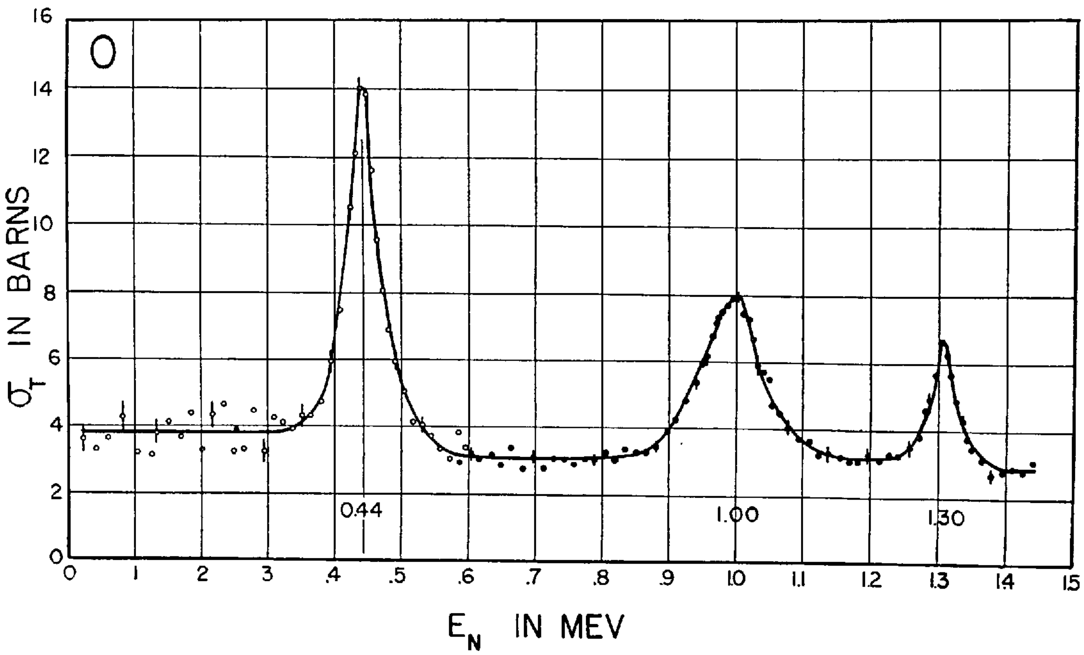
[3] V. F. WEISSKOPF, Helv. phys. Acta *23*, 187 (1950). – E. P. WIGNER, Amer. J. Phys. *17*, 99 (1949).

[4] See for example R. G. SACHS, *Nuclear Theory* (Addison Wesley and Co., 1953), chapter 10, p. 277.

[5] Using neutrons of very low energy, for example those obtained from nuclear reactors, it has been possible to resolve resonances in virtually all heavy elements. These measurements have until recently been restricted to neutron bombarding energies less than a few thousand electron-volts (keV).

[6] H. H. BARSCHALL, Amer. J. Phys. *18*, 535 (1950).

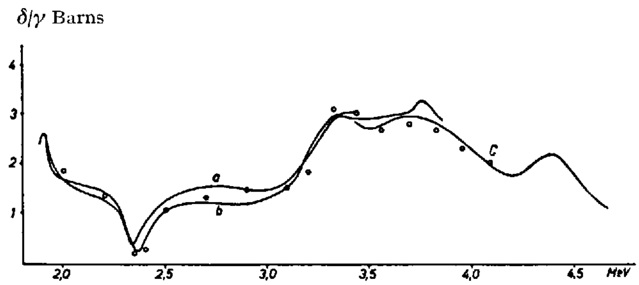
[7] H. H. BARSCHALL, Rev. mod. Phys. *24*, 120 (1952).



a

Fig. 2.—The measured total cross section of oxygen; (*a*) for neutron energies up to 1·5 MeV[6]

that the energy spread ΔE of the neutron beam be small. In particular, to resolve a resonance of width Γ, ΔE should be much less than Γ. An example of a measurement of this kind is given in Figure 2. In part (*a*) of the Figure is shown the measured total cross section of oxygen in an energy region extending from



b

Fig. 2.—The measured total cross section of oxygen; (*b*) for neutron energies between 1·9 and 4·6 MeV[9].

a few kilovolts up to about 1·5 MeV. These cross sections were obtained by the group working under BARSCHALL at Wisconsin[6]. Three resonance levels of the type shown in Figure 1 are in evidence. Since the binding energy of a neutron to an O^{16} nucleus is about 4 MeV, these levels correspond to an excitation of the compound nucleus O^{17} of about 5 MeV. From the heights of the peaks it can be established that the angular momentum in the compound state is $J = 3/2$ units in all three cases. For the resonance at 440 keV, it can be inferred from the width with considerable certainty that the level is formed by neutrons having an orbital angular momentum l equal to one unit[8]. In general, however, it requires a measurement of the angular distribution $\sigma(\vartheta)$ to determine the orbital angular momenta of the neutrons responsible for the excitation of the level. In part (*b*) of Figure 2 is shown the measured total cross section of oxygen from 1·9 MeV to 4·6 MeV. These curves[9] represent results obtained at Wisconsin[8], Zürich[10] and Minnesota[11]. From the fluctuations in the curve it is clear that also in this energy region resonance levels are present. However they are not as clearly separated from each other as those shown in part (*a*) of Figure 2, even though the measurements were made in good resolution. It is therefore more difficult to infer the properties of these levels from a total cross-section measurement. However the anomaly at 2·3 MeV is interesting. The cross-section curve exhibits an inverse peak at this energy, which looks something like the peak of Figure 1 upside down. Actually, for this case equation (3) does not apply, since a term is required corresponding to the interference between the potential scattering σ_0 and the resonant scattering. It is just this destructive interference which is responsible for the inverse peak in Figure 2. Since the destructive interference is so pronounced in this case, it can be shown that the level corresponds to a state formed by neutrons having zero orbital angular momentum and therefore has a total angular momentum ½ unit. In addition, its

[8] C. K. BOCKELMAN, D. W. MILLER, R. K. ADAIR, and H. H. BARSCHALL, Phys. Rev. *84*, 69 (1951).

[9] E. BALDINGER, P. HUBER, and W. G. PROCTOR, Helv. phys. Acta *25*, 142 (1952).

[10] R. RICAMO and W. ZÜNTI, Helv. phys. Acta *24*, 419 (1951).

[11] G. FREIER, M. FULK, E. E. LAMI, and J. H. WILLIAMS, Phys. Rev. *78*, 508 (1949).

width can be determined with reasonable accuracy. This is an exceptional case, however, and angular distribution measurements are required to determine the properties of the other levels shown in Figure 2 (*b*).

Measurements of $\sigma(\vartheta)$ are usually performed in one of two ways. In the first, the element to be investigated is introduced into a chamber in the form of a gas, and the chamber is then inserted into a neutron flux. When a neutron collides with a nucleus of the element under investigation, the recoiling nucleus ionizes the gas, and the charge produced in this ionization can be measured with the help of electrodes inside the chamber. The amount of charge produced is proportional to the recoil energy which in turn depends on the neutron scattering angle, ϑ. By amplifying the charge pulses and performing a pulse-height analysis, the angular distribution can be determined from the pulse-height distribution. This method is, of course, limited to elements which are gases at room temperature, or which occur in simple gaseous compounds. It is further restricted to light elements, since the recoil energy of a heavy nucleus is too small for the charge pulses to be detected above the electronic noise of the system. The other procedure frequently used for determining $\sigma(\vartheta)$ involves placing a solid (or liquid) scattering sample directly in the neutron beam and detecting the neutrons scattered through an angle ϑ by the sample. By varying the position of the neutron detector the dependence of $\sigma(\vartheta)$ on ϑ can be determined.

An example of results obtained using the first of the two procedures described above is shown in Figure 3. HUBER *et al.* at Basle measured $\sigma(\vartheta)$ for oxygen in the energy region from 2 to 4 MeV with an oxygen filled chamber. The experimental data are represented in the figure by the histograms and the smooth curves were calculated from formulas similar to equation (3). From the parameters used in the calculation of the curves giving the best fit, the properties of the levels were inferred.

The example just discussed involved a target nucleus having zero intrinsic spin. This fact considerably simplifies the analysis of the data in terms of the level parameters. For a nucleus such as N^{14} which has spin 1, the interpretation of the total and differential cross-section curves is therefore somewhat less straightforward than for oxygen. In Figure 4 can be seen the total and differential cross sections of nitrogen (as measured by FOWLER and JOHNSON at Oak Ridge[12]) in the energy region between 1·5 and 2·4 MeV. In spite of the added complexity introduced by the nuclear spin in this case, the analysis of the data allows assignments of parameters to the three levels shown in the figure.

In such cases where the interpretation of the data is not unambiguous, measurements of the polarization of the scattered neutrons may permit more definite conclusions to be drawn. In order to measure directly the degree of polarization of neutrons scattered from a given nucleus it is necessary to perform another scattering experiment with these polarized neutrons.

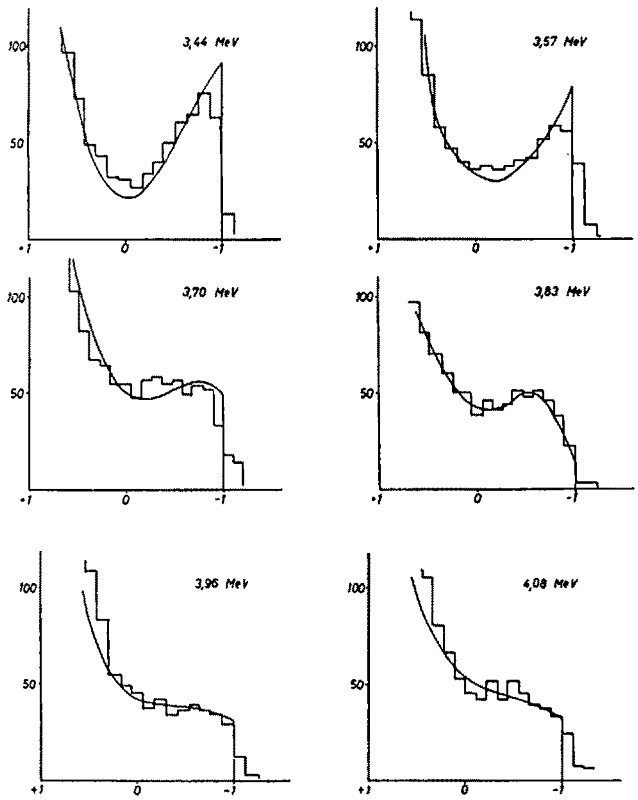


Fig. 3.—The differential elastic scattering cross section $\sigma(\vartheta)$ for oxygen plotted as a function of the cosine of the scattering angle. The histograms represent the experimentally measured values and the smooth curves were calculated using formulas similar to equation (3) generalized for angular distributions. The neutron energy is given with each curve[9].

Since a double scattering experiment of this sort is extremely difficult to carry out, measurements of the polarization produced in scattering are usually made by scattering a beam of polarized neutrons from the nucleus being investigated. In this case the intensity of neutrons scattered to the left through a given angle differs from that scattered to the right through the same angle. The polarization which would be produced in an initially unpolarized beam upon being scattered through this angle can then be inferred from the measured left-right asymmetry, provided the degree of polarization of the incident beam is known.

It is therefore of interest to know the degree of polarization of neutrons produced in various nuclear reactions, aside from the interest which attaches to such polarizations in the study of the reactions themselves. These polarizations can be determined by performing a left-right scattering asymmetry measurement from a sample of an element for which the scattering parameters are sufficiently well known to allow the effect

[12] J. L. FOWLER and C. H. JOHNSON, Phys. Rev. *98*, 728 (1955).

of the scattering nucleus to be calculated. From the measured asymmetry, together with the calculated effect of the scattering nucleus, the degree of polarization of the incident neutrons can be established. This

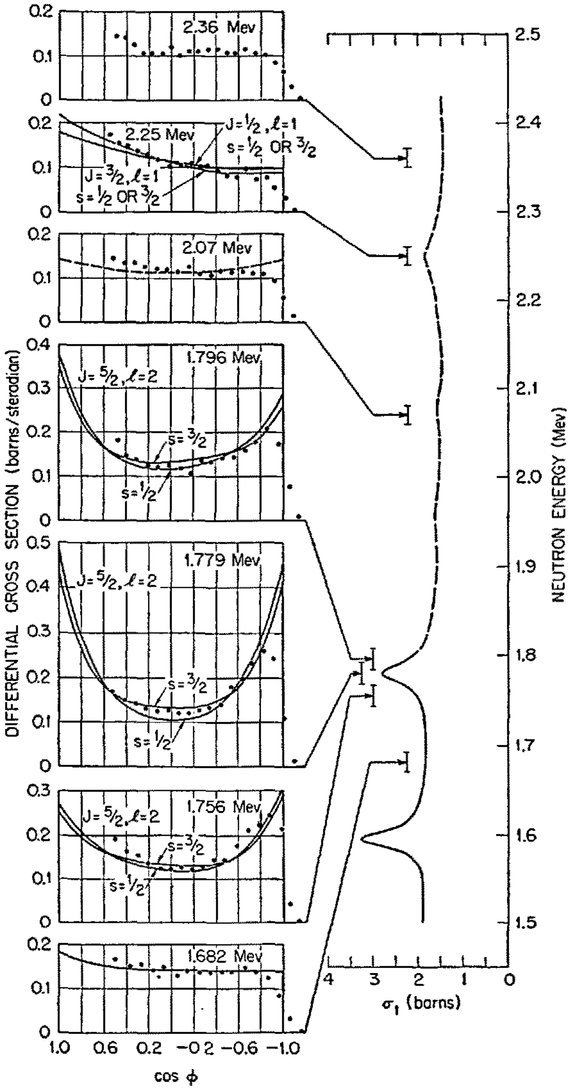


Fig. 4.—Differential and total cross section for the scattering of neutrons by nitrogen. Angular distribution curves are given for several energies between 1·5 and 2·4 MeV. The black dots are data obtained using a nitrogen recoil chamber, and the solid curves represent theoretical calculations using Breit-Wigner type formulas. The parameters used in the calculations are given with the curves; J, total angular momentum of the compound state; l, orbital angular momentum of the neutrons responsible for the resonance scattering; s, channel spin, or sum of neutron spin and spin of N^{14} (s can be either $^3/_2$ or $^1/_2$ in this case). These results do not determine the amount of mixing of the two possible s states[12].

is just the reverse of the procedure described above. It has been used by Baumgartner[13], and by Meier, Scherrer, and Trumpy[14], to investigate the polarization of neutrons emitted from the $D(d,n)He^3$ reaction.

[13] E. Baumgartner and P. Huber, Helv. phys. Acta *26*, 545 (1953).

[14] R. W. Meier, P. Scherrer, and G. Trümpy, Helv. phys. Acta *27*, 577 (1954).

In both experiments the left-right asymmetry in the scattering of neutrons from carbon was measured. In the neutron energy region under consideration, the total and differential cross sections for carbon have been measured and the scattering parameters deduced from these measurements were used to calculate the effect of the scattering on the left-right asymmetry. Observed scattering asymmetries for carbon are shown in Figure 5 along with the asymmetries calculated assuming a degree of polarization of the incident neutrons of 20%.

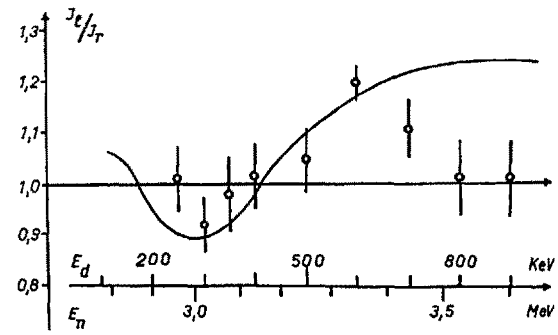


Fig. 5.—Left-right asymmetry of neutrons scattered from carbon plotted as a function of deuteron energy (E_d) and the corresponding neutron energy. The neutrons were emitted at an angle of 45° with respect to the incident deuteron beam, and the scattering angle for the carbon sample was 90°. The smooth curve was calculated from carbon scattering parameters obtained from previous measurements, assuming a degree of polarization of the incident neutrons of 20%[13].

Using the same experimental procedure, but with oxygen as an analyzer, Adair[15] has shown the 400 keV neutrons emitted at 50° from the $Li^7(p,n)Be^7$ reaction to be about 50% polarized.

In summary it may be remarked that determinations of σ_t and $\sigma(\vartheta)$ such as those described above, have been made for virtually all of the available light elements, contributing considerably to the knowledge of nuclear spectroscopy. Interpretation of the level schemes so established in terms of various nuclear models and theories may eventually help lead to a complete understanding of nuclear structure, just as the study of simple atomic spectra helped lead to the understanding of atomic structure.

III. *Intermediate and Heavy Nuclei.* As the number of constituent particles in a nucleus increases, the energy level structure of that nucleus becomes more and more complex. For a given excitation energy, the density of levels is observed in general to increase as one goes to heavier nuclei, and the widths of the levels are observed in general to decrease. As a result it becomes more difficult to perform cross-section measurements in good resolution, to say nothing of the increased complexity of the situation from a theoretical standpoint. For these reasons it is simpler to perform

[15] R. K. Adair, S. E. Darden, and R. E. Fields, Phys. Rev. *96* 503 (1954). – A. Okazaki, Phys. Rev. *99*, 55 (1955).

average measurements on heavy nuclei. Average measurements are those performed with neutron beams having an energy spread ΔE much larger than the average spacing between levels of the compound nucleus. This is just the opposite of the situation encountered in studying light nuclei, where ΔE is usually much less than the widths of the levels. From the experimental standpoint, such average measurements possess several advantages. For one thing, the target used for producing the neutrons via a nuclear reaction can be relatively thick, i.e. contain many atoms of the element A [equation (1)]. This means an increased probability for the reaction, therefore a higher intensity of the neutron beam with a consequent reduction in the time required to perform a measurement. Furthermore, the requirements on the constancy of the energy of the bombarding charged particles are not so stringent.

A schematic theory which attempted to explain and predict the behavior of average cross sections was developed by WEISSKOPF *et al.* at M.I.T.[16]. This continuum theory, as it was called, was based upon BOHR'S model of the compound nucleus. It was assumed that once a neutron succeeded in penetrating to the interior of a nucleus, it would share its energy with the constituent nucleons and form a compound nucleus as discussed in Section I. This excited compound nucleus would then decay with the emission of some nuclear particle (or particles), or a gamma ray, or both. In any case, it was postulated, the probability that the compound nucleus would decay by emitting a neutron having the same energy as the bombarding neutron, i.e. the reversal of the formation process, should be negligibly small, since there are presumably so many other processes which could take place. The continuum theory assumed further that for the purposes of describing the average neutron-nucleus interaction, the nucleus could be considered to be a sphere, the radius of which depends smoothly on the atomic weight A according to the formula

$$R = r_0 A^{1/3}, \qquad (5)$$

where r_0 is of the order of 1.5×10^{-13} cm. By formulating these assumptions quantitatively, WEISSKOPF and co-workers were able to predict the average properties of the neutron-nucleus interaction as a function of both atomic weight and energy. In particular the continuum theory predicted the average total cross section $\bar{\sigma}_t$, the average non-elastic cross section $\bar{\sigma}_{in}$, the average differential elastic scattering cross section $\bar{\sigma}(\vartheta)$, and a quantity called the average neutron width-to-spacing ratio $\bar{\Gamma}nl/\bar{D}$. The last mentioned quantity is just the average neutron width for levels of the compound nucleus formed by neutrons having orbital angular momentum l units, divided by the average spacing between such levels. An important point of this theory was that these average properties were expected to vary smoothly with atomic weight and neutron energy. For example, the square root of the average non-elastic cross sections measured at a fixed neutron energy were expected to increase linearly with the cube root of the atomic weight. Some measured non-elastic cross sections are shown in Figure 6[17] along with the predictions of the continuum theory. The neutron energy for these measurements was 14 MeV. In this case, the agreement between theory and experiment is quite good.

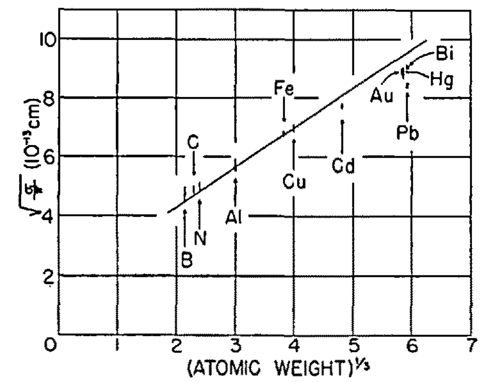


Fig. 6.—Square root of measured non-elastic cross sections plotted *versus* the cube root of the atomic weight. The neutron energy is 14 MeV. The straight line is predicted by the continuum theory[17].

The average total cross sections, according to the continuum theory, should decrease monotonically with increasing neutron energy and neighboring elements should exhibit practically identical behavior. A survey of average total cross sections[18] was made at Wisconsin in 1951 for several heavy elements in the neutron energy region between about 100 keV and 3 MeV. The results of these and several additional measurements are shown in Figure 7[19]. For convenience the total cross section divided by the "nuclear area" πR^2 is plotted versus neutron energy and atomic weight. R was computed from equation (5) using $r_0 = 1.45 \times 10^{-13}$ cm. It can be seen that the smooth dependence on energy and atomic weight expected from the theory is substantiated by the results. However the cross sections do not all decrease monotonically with increasing neutron energy but instead exhibit bumps in certain energy and atomic weight regions. This disagreement between theory and experiment led to a modification of the continuum theory which resulted in the complex potential well model[20]. This description of the average

[16] H. FESHBACH, D. C. PEASLEE, and V. F. WEISSKOPF, Phys. Rev. *71*, 145 (1947). – H. FESHBACH and V. F. WEISSKOPF, Phys. Rev. *76*, 1550 (1949).

[17] H. H. BARSCHALL, Amer. J. Phys. *22*, 517 (1954).

[18] D. W. MILLER, R. K. ADAIR, C. K. BOCKELMAN, and S. E. DARDEN, Phys. Rev. *88*, 83 (1952).

[19] *Statistical Aspects of the Nucleus*, Report of a conference held at Brookhaven National Laboratory in January 1955.

[20] H. FESHBACH, C. E. PORTER, and V. F. WEISSKOPF, Phys. Rev. *96*, 448 (1954).

neutron-nucleus interaction differs from the continuum theory in that it represents the neutron-nucleus interaction by a potential of the form

$$V = -V_0\,(1+i\,\zeta) \quad r < R, \qquad = 0 \quad r > R. \tag{6}$$

Here r is the distance between the neutron and the center of the nucleus, and R is given by equation (5). There are three parameters in this schematic theory; V_0, the depth of the well; ζ, the magnitude of the imaginary part; and r_0, the nuclear radius constant. It has been found that the measured total cross sections shown in Figure 7 can be reasonably well reproduced by assigning the following values to the parameters

$$V_0 = 42\,\mathrm{MeV}, \quad \zeta = 0{\cdot}03, \quad r_0 = 1{\cdot}45 \times 10^{-13}\,\mathrm{cm}.$$

Cross sections calculated using these parameters are shown in Figure 8. The agreement between these curves and those of Figure 7 is remarkable when one considers that only three parameters are involved. One can understand qualitatively the features of the total cross sections on the basis of this model by considering the potential of equation (6). Ignoring the imaginary part, it can be seen that the interaction is that of a particle moving in an attractive force field. Since the neutrons have a wave-like nature they are refracted when they reach the surface of the potential, some of them being reflected and some penetrating to the inside of the sphere. When these waves inside the sphere reach the other side they will again be refracted and reflected and this will continue back and forth. It is clear that if the wave length of the neutron inside the well has the proper value the various waves scattered from the nucleus can interfere constructively, giving rise to peaks or bumps in the cross sections like those in Figures 7 and 8. The inclusion of an imaginary term in the potential has the effect of damping the waves inside the nucleus and this damping reduces the intensity of the waves being reflected back and forth inside the sphere. This decrease in intensity of the neutron waves inside the sphere is interpreted as corresponding to the absorption of neutrons in the nucleus. By absorption here is meant that the neutrons are removed from coherence with the incident beam and can no longer interfere with it. This absorption of neutrons can then lead to the non-elastic processes. If there were no imaginary part in the potential then there would be no absorption of neutrons inside the nucleus and the interaction would consist entirely of elastic scattering. The fact that a value of ζ of only 0·03 is indicated by the experimental data of Figure 7 suggests that the absorption of neutrons having an energy of 1 MeV or so is fairly small, or in other words, the neutron has a rather good chance of penetrating the nucleus and being refracted out again without being absorbed. It is in just this respect that the complex square well model mostly differs from the continuum theory, where it was assumed that compound nucleus formation always followed penetration of the neutron into the nucleus.

The interaction of neutrons with a complex square well is exactly analogous to the refraction of light waves by a sphere of refracting material having a complex index of refraction, i.e. a sphere which both refracts and absorbs light. For this reason the model has also been called the optical model[21], or the cloudy crystal ball model. It must be emphasized that this description of the neutron-nucleus interaction is expected to apply only to the average interaction and as such should not be taken as a literal description of what happens when a neutron collides with a nucleus.

The angular distributions of elastically scattered neutrons predicted by the complex well model will of course differ from those predicted by the continuum theory. Whereas with the continuum theory the angular distributions are determined by the reflection and diffraction scattering of the neutrons from the heavily absorbing nucleus, with the cloudy crystal ball model elastic scattering can take place in two ways. The first way is the refraction or shape elastic scattering brought about by the shape of the well. On the other hand, since it is possible for a compound nucleus to be formed by the absorption of a neutron, this compound nucleus may decay with the emission of a neutron having the same energy as the incident neutron. Such a process is called compound elastic scattering and is experimentally indistinguishable from the shape elastic scattering. The amount of this latter type of elastic scattering taking place will clearly depend on the various ways in which the compound nucleus can decay. If the neutron energy is so low, for example, that no other processes such as reactions or inelastic scattering of the neutron can occur, then the compound nucleus will always decay in such a way as to give rise to compound elastic scattering. In this case the non-elastic cross section will be zero. When the compound nucleus can decay in so many different ways that the chances of compound elastic scattering taking place are negligible, then the non-elastic cross section is expected to have its maximum value allowed by the theory and is equal to the so-called cross section for formation of the compound nucleus. It should be pointed out that the cross section for formation of the compound nucleus refers to all processes in which the neutron is absorbed by the nucleus. Physically, this can occur in a number of ways and need not involve the formation of the compound nucleus in the Bohr sense of the word. For example, the neutron may collide directly with a single nucleon in the nucleus and eject it. Another possibility

[21] It should be pointed out that this is not the first application of this model to nuclear scattering problems. It has also been used to describe the scattering of high energy nucleons from nuclei as well as the scattering of mesons from nuclei.

is that the neutron may excite collective oscillations of the nucleus which give rise to nuclear surface waves. This motion is of a somewhat specialized sort and does not correspond to the chaotic conditions usually associated with a Bohr type compound nucleus. All such processes are also included in the cross section for formation of the compound nucleus and a separation of this cross section into partial cross sections for the various processes requires a more detailed theory.

From what has been said it is clear that the potential of equation (6) can only predict upper and lower limits for $\sigma(\vartheta)$ and an upper limit for the non-elastic cross section. What the model does predict is the shape elastic scattering, and the sum of the compound elastic scattering and the non-elastic scattering and this sum is just the cross section for compound nucleus formation.

In order to further test the cloudy crystal ball model, many measurements of $\overline{\sigma}(\vartheta)$ and the average non-elastic cross sections have been made for intermediate and heavy elements. Values of $\sigma(\vartheta)$ obtained by Walt and Barschall[22] for neutrons of 1 MeV energy are shown in Figure 9. These data were taken using the second of the two methods discussed in Section II for measuring angular distributions. The energy spread of the neutron beam amounted to about 100 keV which is much greater than the level spacing of the compound nucleus for most of the elements represented in Figure 9. Just as in the case of the total cross sections, the angular distributions exhibit a smooth dependence on atomic weight. A similar plot of the predictions of the complex well model is given in Figure 10, where the

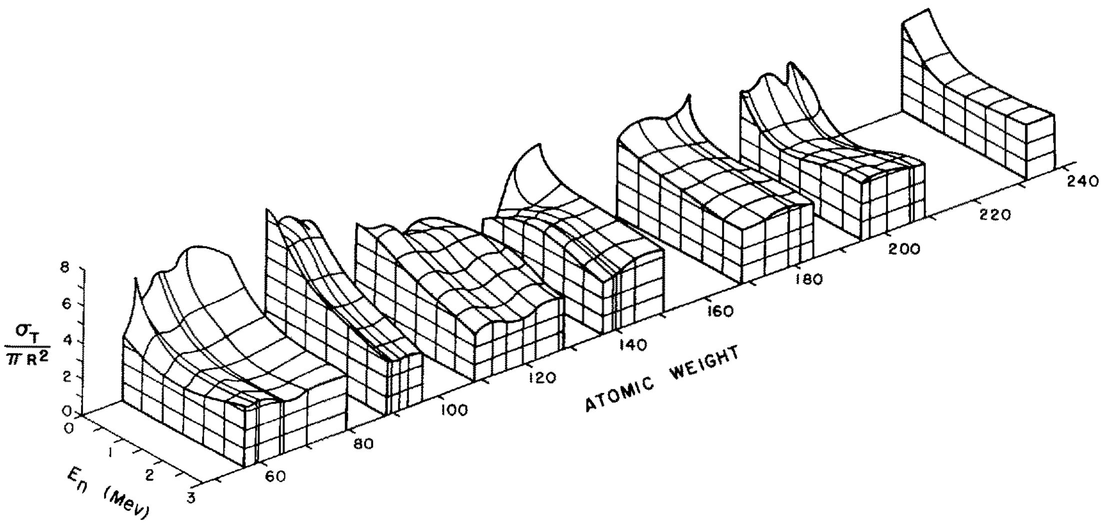


Fig. 7.—Measured total cross sections plotted as a function of atomic weight and neutron energy[19].

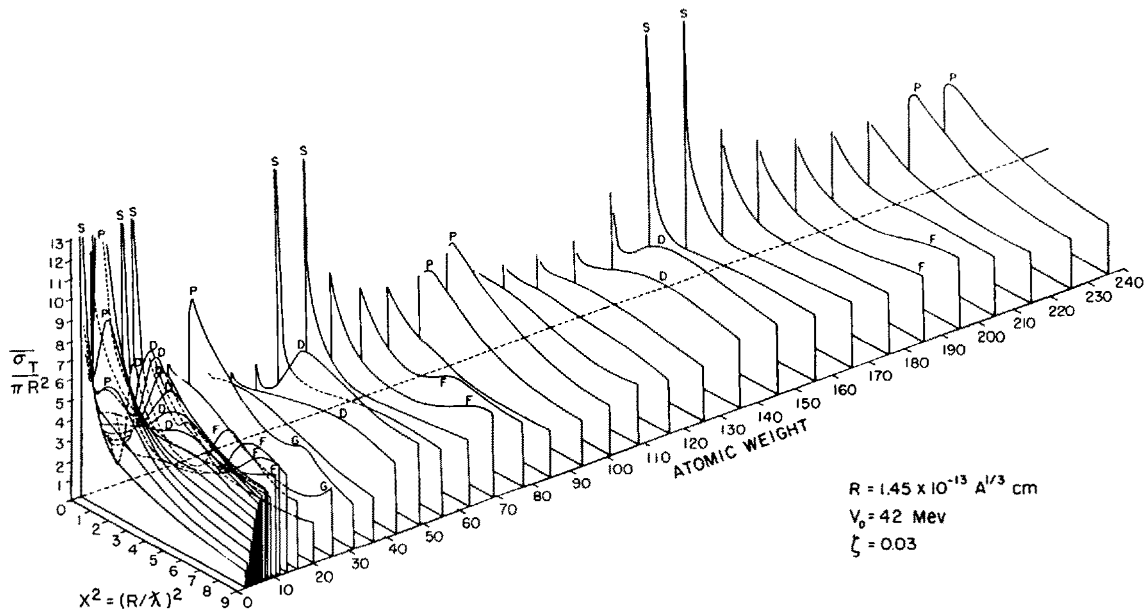


Fig. 8.—Average total cross sections calculated from the potential of equation (6)[19].

[22] M. Walt and H. H. Barschall, Phys. Rev. 93, 1062 (1954).

shape elastic scattering and the maximum amount of compound elastic scattering allowed by the model are included in the curves. A comparison of Figures 9 and 10 indicate qualitative agreement between theory and experiment, although some discrepancies are in evidence in the atomic weight region near 200. It is perhaps not surprising that the measured angular distributions seem to agree somewhat less well with the theory than the total cross sections, since the former test the theory more strongly than the latter. The differential cross sections shown in Figure 9 were integrated over the solid angle and subtracted from the measured total cross sections in order to obtain the average non-elastic cross sections. In Figure 11 the values of $\bar{\sigma}_{in}$ so obtained are compared with the theoretical cross sections for formation of the compound nucleus, which, as we have seen, give an upper limit for $\bar{\sigma}_{in}$. The peaks in the measured values of $\bar{\sigma}^{in}$ are seen to fall at the wrong atomic weights as far as the theory is concerned, although the situation can be improved by changing some of the parameters in the theory.

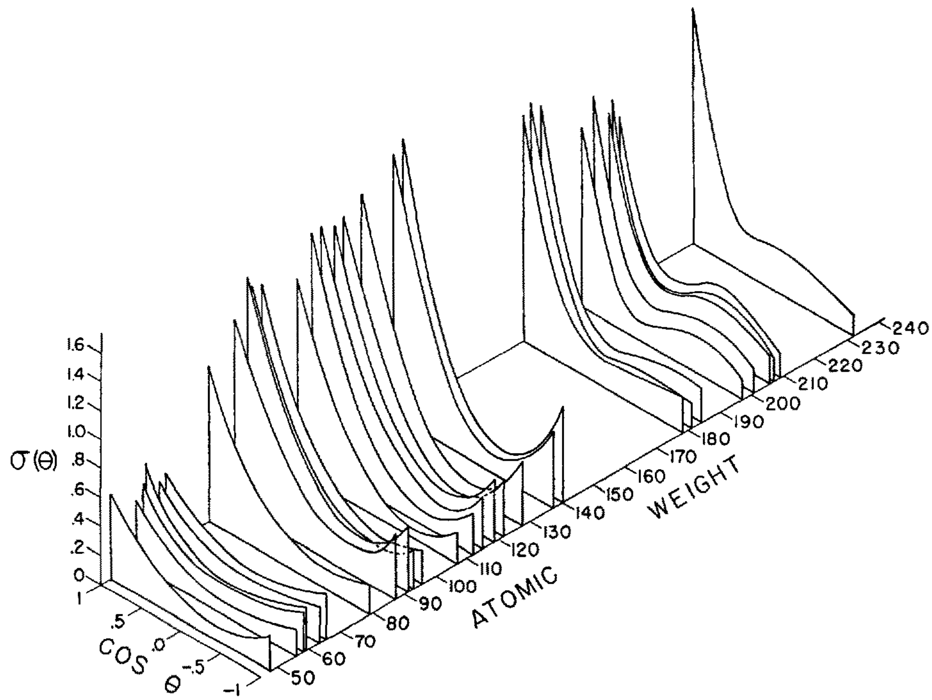


Fig. 9.—Measured average differential elastic scattering cross sections for 1 MeV neutrons, plotted *versus* cos ϑ and atomic weight[22].

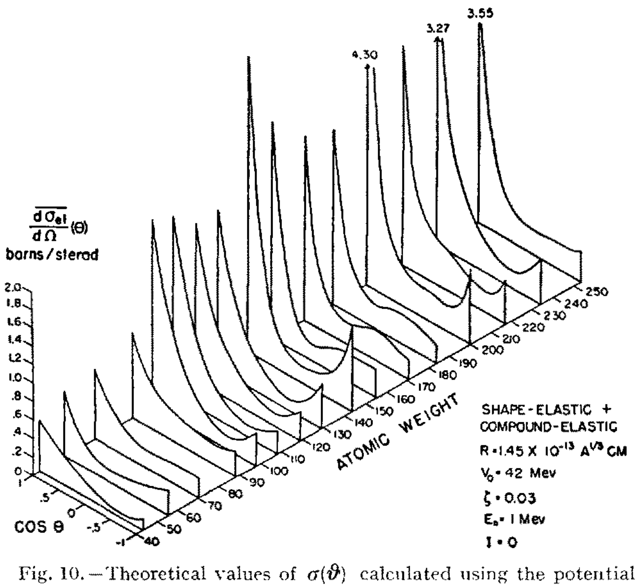


Fig. 10.—Theoretical values of $\sigma(\vartheta)$ calculated using the potential of equation (6). The curves include the contribution of the shape elastic scattering and the maximum possible amount of compound elastic scattering[20].

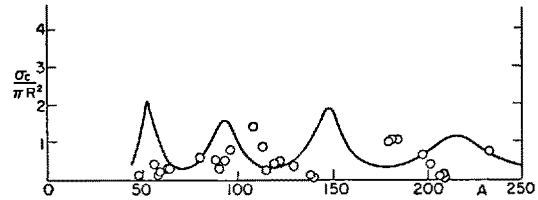


Fig. 11.—Non-elastic cross sections σ_{in} for 1 MeV neutrons as determined by WALT and BARSCHALL. The circles are the measured values of σ_{in} divided by the 'nuclear area' πR^2. The abcissae are atomic weights. The smooth curve gives the calculated cross section for compound nucleus formation[20].

Angular distribution measurements at higher neutron energies also show qualitative agreement with the predictions of the cloudy crystal ball model although certain modifications of the model are called for. It has become evident that at a neutron energy of 4 MeV, for example, a larger imaginary part of the potential is required to fit the calculated angular distributions to the measured values. This is exemplified in Figure 12, where measured angular distributions[23] for Zr, Sn, Ta,

[23] M. WALT and J. R. BEYSTER, Phys. Rev. *98*, 677 (1955).

and Pb are shown together with calculated curves for $\bar{\sigma}(\vartheta)$. Curves computed using three values of the parameter ζ in equation (6) are shown and it is clear that a value of ζ between 0·1 and 0·2 is required at this energy rather than the value 0·03 which best fits the total cross section data at lower energies. A similar situation is encountered when the measured non-elastic cross sections[24] at 4 MeV are compared with the calculated cross sections for compound nucleus formation. For some elements the observed values of $\bar{\sigma}_{in}$ are such that even with the relatively large amount of absorption associated with a value of ζ of 0·2 (mean free path of the neutron in nuclear matter about equal to the radius of a nucleus of atomic weight 200) these observed values of $\bar{\sigma}_{in}$ are greater than the computed cross sections for compound nucleus formation. It has accordingly been found necessary to change the model in such a way as to allow for more absorption of the neutrons by the nucleus at higher neutron energies. This has been accomplished by rounding off the side of the potential well, which decreases the amount of reflection of the neutron waves at the nuclear surface and thereby enhances the probability of absorption inside the well. This is again analogous to the optical case, in which an extremely sharp boundary between two media reflects more strongly than a diffuse boundary. With a rounded potential of this sort it appears to be possible to substantially improve the agreement between measured and calculated cross sections[25].

The increase in the probability of neutron absorption with increasing neutron energy is a trend toward the type of interaction postulated by the continuum theory. In view of this, it is not surprising that the measured non-elastic cross sections at 14 MeV shown in Figure 6 agree fairly well with the continuum theory. The increased neutron absorption at higher neutron energies has been interpreted[26] as being a consequence of the Pauli principle. At low energies, relatively few collisions between the incident neutron and the constituent nucleons of the nucleus can take place, since the recoil nucleons would in most cases end up in states that are already filled. As the neutron energy increases, however, more and more final states become available with the result that the chances for a collision, and therefore the absorption, rises.

Another modification which has increased the agreement between experiment and theory is the introduction of a slightly different expression for the nuclear radius than that of equation (5). The formula which has been suggested[19] is

$$R = (1{\cdot}26\, A^{1/3} + 0{\cdot}76)\, 10^{-13}\ \text{cm.} \qquad (7)$$

[24] J. R. Beyster, R. L. Henkel, R. A. Nobles, and J. M. Kister, Phys. Rev. *98*, 1216 (1955).
[25] E. W. Salmi, J. R. Beyster, and M. Walt, Bull. Amer. phys. Soc., Series II, Vol. I, No. 4, 194 (1956).
[26] A. M. Lane and C. F. Wandel, Phys. Rev. *98*, 1524 (1955).

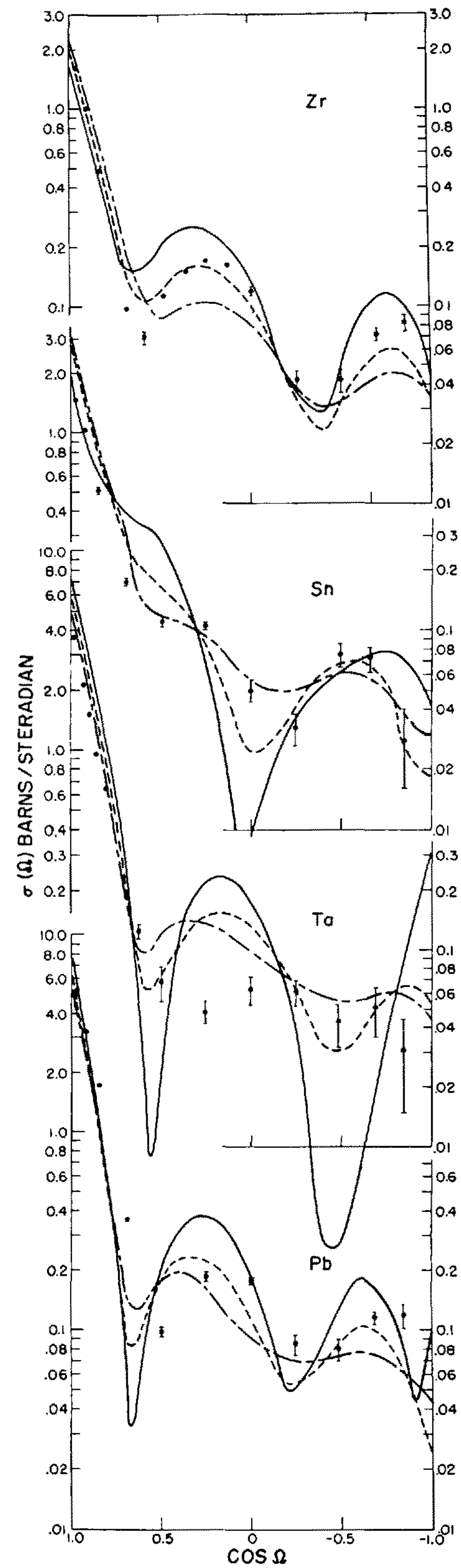


Fig. 12.—Comparison of theory and experiment at a neutron energy of 4·1 MeV for Zr, Sn, Ta, and Pb. The experimental points were measured by Walt and Beyster[23] at Los Alamos. The curves are differential shape-elastic scattering cross sections calculated using the potential of equation (6). The solid line, broken line, and dot-and-dash line are for values of ξ = 0·03, 0·1, and 0·2 respectively. The abscissa is the cosine of the scattering angle ν.

Radii calculated from equation (7) are smaller for the heavier nuclei than those calculated from equation (5), a change which moves the theoretical curve in Figure 11 somewhat to the right for larger A values and brings the peaks in the curve more nearly into coincidence with the maxima of the experimental values.

It has been pointed out above that a schematic theory of the average neutron-nucleus interaction makes predictions concerning the behavior of $\Gamma_{nl}/\overline{D}$, the average neutron width-to-spacing ratio. For an individual resonance, Γ_{nl} is a measure of the probability that the compound state in question will be formed when a target nucleus is bombarded by neutrons having energies near E_r. This width can be written as a product of two factors:

$$\Gamma_{nl} = F_l \ \gamma_l^2. \tag{8}$$

The first factor expresses the neutrons' chances of penetrating the centrifugal potential barrier and arriving at the nuclear surface. This factor accordingly depends only on the neutron energy and nuclear size and can be calculated. The second factor is a characteristic of the compound nuclear state and gives the probability that a neutron at the surface of the target nucleus will combine with the nucleus in such a way as to form the compound nuclear state in question. Since the F_l factors can be calculated, the behavior of $\overline{\Gamma_{nl}/D}$ can be equally well given by the behavior of $\overline{\gamma_l^2/D}$. This quantity is referred to as the strength function. Since γ_l^2 is proportional to the probability for forming a compound nuclear state, it is evident that the strength function $\overline{\gamma_l^2/D}$ is a measure of the cross section for formation of the compound nucleus by neutrons of orbital angular momentum l. According to the complex square well model, the strength function should exhibit maxima and minima as a function of neutron energy and atomic weight just as the average cross sections do. In fact the peaks in $\overline{\gamma_l^2/D}$ are expected to essentially coincide with the peaks in the total cross section curves. This is again similar to the optical case, in which the dispersion and absorption curves for a substance having a complex index of refraction both show anomalies at the same wave lengths. The size of these maxima in $\overline{\gamma_l^2/D}$ depends strongly on the imaginary part of the complex well. For small values of $\overline{\zeta}$ (e.g. 0·03), the peaks in $\overline{\gamma_l^2/D}$ are high and narrow. As the amount of absorption increases with increasing $\overline{\zeta}$ the peaks become lower and broader until they finally disappear and $\overline{\gamma_l^2/D}$ is everywhere the same, a situation which corresponds to the continuum theory.

At very low neutron energies where measurements of σ_t in good energy resolution are available, values of γ_l^2 and D can be obtained directly by fitting Breit-Wigner formules to the experimental cross section curves. A summary of results on γ_0^2/D obtained by averaging γ_0^2 and D over several levels of each nucleus involved are shown in Figure 13[19]. The cross sections from which these values were obtained were measured at neutron energies of a few kilovolts and less, using neutrons from nuclear reactors. In view of the low neutron energy only neutrons having zero orbital angular momentum interact appreciably. The curve in Figure 13, which shows the predictions of the cloudy crystal ball model, reproduces the experimental results qualitatively, although the agreement could be better for the heavy elements. As with the data on $\overline{\sigma}_{in}$ at 1 MeV, the agreement here would be improved by using radii calculated from equation (7).

Another way of measuring the strength function makes use of the fact that the presence of resonances in the total cross section of an element produces a small deviation from exponential attentuation of a neutron beam passing through a sample of the element. This means that a small correction term must be added to the right hand side of equation (4) if the energy spread of the neutron beam is broad enough to include resonances. The magnitude of this correction term can be shown to depend on $\overline{\gamma_l^2/D}$[27]. Results obtained with this method are qualitatively what one expects from the potential of equation (6), but the smallness of the effects limits the accuracy of the method[28].

It was mentioned in Section I that investigations of the polarization produced in a beam of neutrons upon being scattered from a nucleus may lead to information concerning the forces responsible for the scattering. No allowance is made for the relative orientation of neutron spin and orbital angular momentum in the potential of equation (6). From what is known about the ground states, or lowest energy levels of nuclei, however, it is clear that such forces play a role in determining the relative positions of the levels. This is essentially the same situation which obtains in the fine structure splitting of atomic energy levels, although the origin of the forces may not be the same in the two cases. It is possible that these spin-orbit forces, as they are called, influence the scattering of fast neutrons from nuclei even when the energy spread of the neutron beam averages over many levels of the compound nucleus. To investigate this point it would be desirable to conduct a systematic study of the polarization produced in scattering as a function of neutron energy, scattering angle, and atomic weight. Unfortunately the experimental difficulties involved in carrying out such measurements are considerable. Determinations of polarization produced in scattering are usually made by scattering a beam of neutrons posessing a known degree of polarization from a sample of the nucleus being studied as discussed in section II. A measurement of the left-right asymmetry in the scattering then

[27] R. G. THOMAS, Phys. Rev. *98*, 77 (1955).

[28] S. E. DARDEN, Phys. Rev. *99*, 748 (1955). – H. H. BARSCHALL and S. E. DARDEN, Phys. Rev. *100*, 1242 (1955).

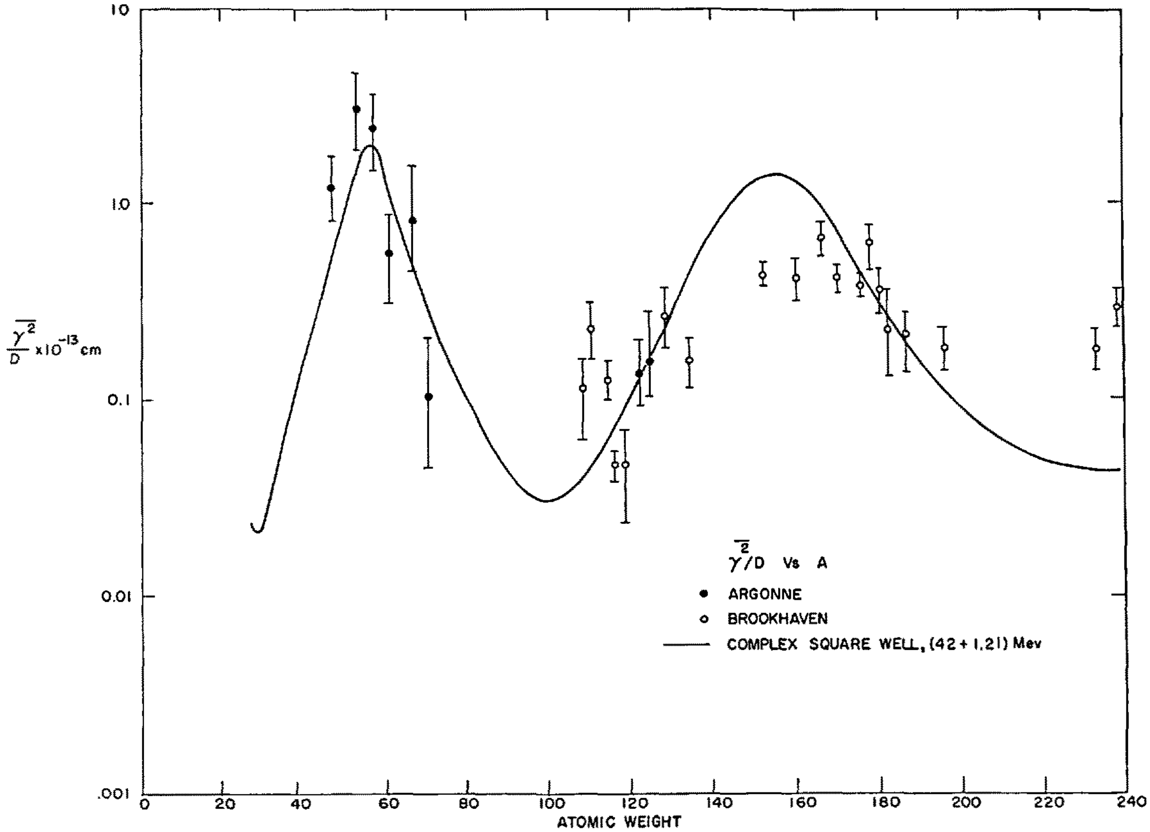


Fig. 13.—$\gamma_0^2/\bar{D}$ plotted as a function of atomic weight. The experimental points were obtained from neutron fast chopper data. The curve was calculated from the complex square well potential[19].

suffices to establish the degree of polarization which would be produced in the scattering of an unpolarized beam from the sample. For only a very few reactions

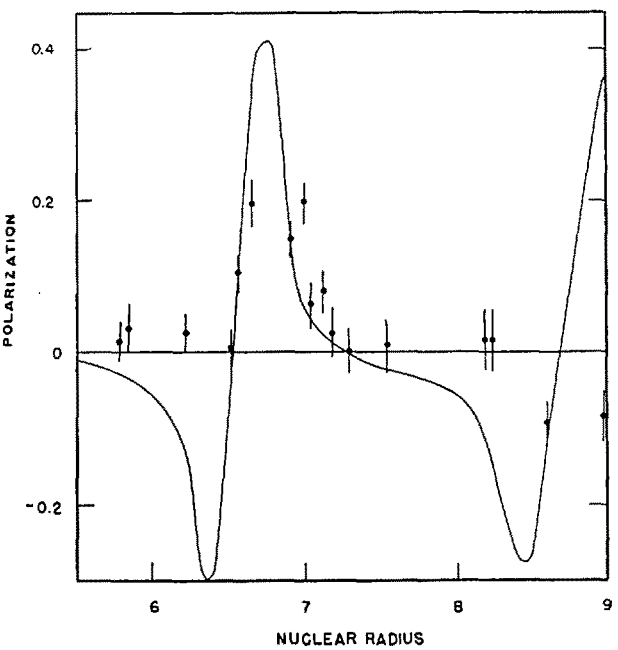


Fig. 14.—Polarization produced in the scattering of 400 keV neutrons through 90° by various elements. The abcissae are nuclear radii. The curve was calculated from the complex square well modified by the addition of a spin orbit term[15].

is the degree of polarization of the emitted neutrons sufficiently well known to make an experiment of the kind just described feasible. One such reaction is the $Li^7(p,n)Be^7$ reaction discussed in Sections I and II. Experiments have been carried out in which neutrons produced in this reaction were scattered from a number of heavy elements[15]. From the observed left-right asymmetries, polarizations produced in scattering from these elements were determined. The results are plotted in Figure 14. Polarizations are plotted against nuclear radii calculated using equation (5) with $r_0 = 1{\cdot}45 \times 10^{-13}$ cm. The solid curve was computed from the potential of equation (6) modified by the addition of a spin-orbit term

$$V^1 = -3\ \text{MeV}\ (\vec{L} \times \vec{S}), \tag{9}$$

where $\vec{L}$ is the neutron orbital angular momentum vector and $\vec{S}$ is the neutron spin vector. This has the effect of making the well deeper than in equation (6) for neutrons having their spin and orbital angular momentum parallel, and shallower for those with antiparallel spin and orbital angular momentum. It is not clear from Figure 14 that the interaction responsible for the positive polarizations observed for nuclei having radii near 7×10^{-13} cm is the same as that of equation (9), since the large negative polarizations expected from such an interaction are not observed.

Some similar measurements[29] using neutrons of higher energy from the D^2 $(d,n)He^3$ reaction have been performed. These indicate qualitative agreement with the polarizations calculated assuming a spin-orbit interaction of the type in equation (9). As yet however, the data are so sparse and the experimental uncertainties so large that little can be said about the spin-orbit forces in the average neutron-nucleus scattering interaction.

[29] A. E. REMUND, Helv. phys. Acta, *29*, 545 (1956).

Zusammenfassung

Die Untersuchung der Atomkerne mit Neutronen wird diskutiert. Bei den leichten Kernen geben die Experimente Auskunft über die Energieniveaux des betreffenden Kerns. Die Experimente an schweren Kernen lassen sich am einfachsten an Hand einer gemittelten Kern-Neutron-Wechselwirkung interpretieren.

Brèves communications - Kurze Mitteilungen Brevi comunicazioni - Brief Reports

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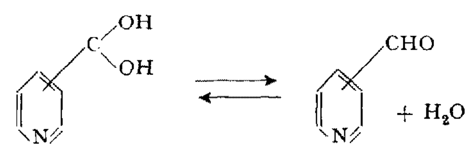
Polarographic Reduction of Isomeric Pyridine-Aldehydes at the Dropping Mercury Electrode

In acidic supporting electrolytes, e.g. solutions of strong acids (0·1 N H_2SO_4, 0·1 N HCl) or buffer solutions of pH 1–2, all three isomeric pyridine-monoaldehydes give polarographic waves which are much lower than the full diffusion current corresponding to the reduction of aldehydes according to the Ilkovič equation. With pyridine-3-aldehyde at pH 1·9, the height of the wave is about 19% of the current due to a two-electron process. The limiting current in this region does not depend on the height of the mercury column (Fig. 1). With increasing pH, the height of the waves grows until it reaches a limiting value for all three aldehydes at the pH of about 6.

The wave of pyridine-3-aldehyde is simple, while the waves of pyridine-2-aldehyde and pyridine-4-aldehyde are composed of two parts of approximately equal height. Simultaneously with the growth of the waves with pH, the waves gradually attain the characteristics of a diffusion current. In the alkaline range of pH, the diffusion current remains nearly constant and decreases only at pH greater than 10 (Fig. 2).

The reduction proceeds in the case of pyridine-4-aldehyde (pH 4) at – 0·35 V and – 0·49 V vs. N.C.E., of pyridine-2-aldehyde at – 0·56 V and of pyridine-3-aldehyde at – 0·79 V. The half-wave potentials of all three compounds are shifted to more negative values with increasing pH, the slope being 60 mV/pH for pyridine-3-aldehyde and 75 mV/pH for the other two.

From the variation of the height of the waves with the concentration of hydrogen ions, which is similar to that of formaldehyde[1], it can be assumed that in aqueous medium the three compounds are present mainly in their hydrated form, the concentration of the free aldehyde being low. Only the latter is polarographically reducible. The limiting currents are controlled by the rate of dehydration of the hydrated form as the two forms are in a reversible equilibrium:



This reaction is subject to general base catalysis, hence the height of the waves varies with the concentration of

[1] K. VESELÝ and R. BRDIČKA, Coll. Trav. chim. Tchécosl. *12*, 313 (1947). – R. BIEBER and G. TRÜMPLER, Helv. chim. Acta *30*, 706 (1947). – R. BRDIČKA, Chem. Listy *48*, 1458 (1954).

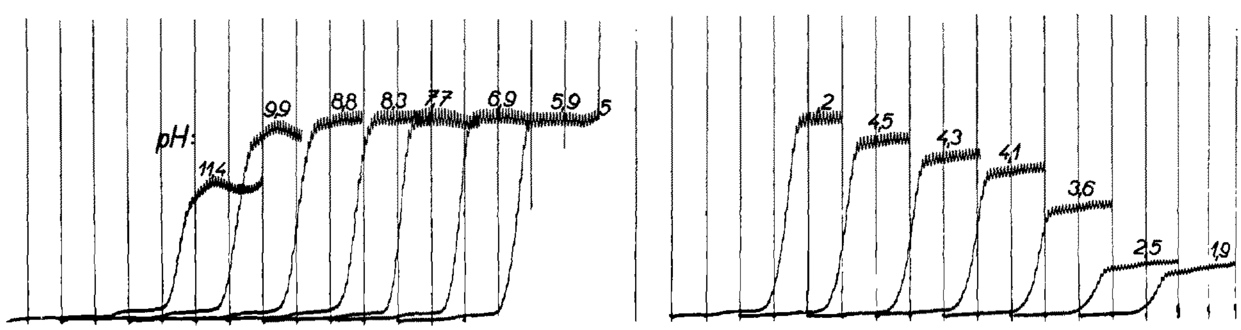


Fig. 1.—Dependence of the wave of pyridine-3-aldehyde on the pH of the supporting electrolyte.
Electrolyte: buffer solution according to BRITTON and ROBINSON; 1·12 × 10^{-3} M pyridine-3-aldehyde; from – 0·2 V (satd. cal. elect.); 196 mV/absc.; nitrogen atm.; s = 1:30.